Current stage of understanding and description of hadronic elastic diffraction

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Abstract

Current situation with phenomenological description of high-energy nucleon-nucleon diffractive elastic scattering is reviewed. Comparison of various model predictions with the recent D0 and TOTEM data on the nucleon-nucleon total and differential cross-sections is presented.

Introduction

Diffractive phenomena in hadron physics are related to strong interaction. QCD is recognized as the fundamental theory of strong interaction. Thus, description of diffractive reactions at high energies should be grounded on some QCD-based techniques. But the special status of diffractive studies at high-energy colliders is determined by the fact that diffraction of hadrons takes place due to interaction at large distances. Indeed, the transverse size of the hadron interaction region can be estimated through the corresponding Heisenberg uncertainty relation, and extraction of this quantity from experimental elastic angular distributions can be done without dealing with any theory. For example, at the SPS, Tevatron, and LHC energies it is of order 1 fm. Technically, this means that we are in the so-called "non-perturbative regime" and straight applying of QCD to description of hadronic diffraction is disabled, since QCD, at its current stage of development, has no essential progress outside of perturbative calculations.

Hence, one is enforced to invent "plausible" models which bear, at least, general QCD properties, as much as possible.

Scattering amplitude, Born term ("eikonal") and Regge trajectories

In the vast majority of diffraction models there is used the notion of reggeons (analytic continuations of resonance spectra).

Below, the recipe for calculation of the elastic scattering amplitude in the framework of the Regge-eikonal approach [1] is presented:

$$T_{12\to 12}(s,t) = 4\pi s\int_0^\infty db^2 J_0(b\sqrt{-t})\frac{e^{2i\delta_{12\to 12}(s,b)}-1}{2i}\,,$$

$$\delta_{12\to 12}(s,b) = \frac{1}{16\pi s}\int_0^\infty d(-t)J_0(b\sqrt{-t})\delta_{12\to 12}(s,t) = \frac{1}{16\pi s}\int_0^\infty d(-t)J_0(b\sqrt{-t})\times$$

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$$\times\left\{\sum_n\left(i+\mathrm{tg}\frac{\pi(\alpha_n^+(t)-1)}{2}\right)\Gamma_n^{(1)+}(t)\Gamma_n^{(2)+}(t)s^{\alpha_n^+(t)}\mp\right.$$
$$\left.\mp\sum_n\left(i-\mathrm{ctg}\frac{\pi(\alpha_n^-(t)-1)}{2}\right)\Gamma_n^{(1)-}(t)\Gamma_n^{(2)-}(t)s^{\alpha_n^-(t)}\right\}.$$

Here s and t are the Mandelstam variables, b is the impact parameter, T is the elastic scattering amplitude, eikonal δ is the sum of single-reggeon-exchange terms, $\alpha(t)$ are Regge trajectories, and $\Gamma(t)$ are reggeonic form-factors of colliding particles. Besides the Regge-eikonal approximation, there exist various approaches exploiting the notion of reggeons.

Calculation of Regge trajectories within QCD is one of the main theoretical problems of hadron physics. What are fundamental achievements in this direction?

Regge trajectories and QCD

The first approach yielding some results on QCD Regge trajectories is the famous BFKL approach based on solving the so-called BFKL equation which is some modification of the Bethe-Salpeter equation. Within this approach, behavior of Regge trajectories for reggeons composed of two reggeized partons was calculated at asymptotically high transfers (in the case of quark-antiquark pair by Kwiecinski [2] and for two gluons by Kirschner and Lipatov [3]):

$$\alpha_{\bar{q}q}(t)=\sqrt{\frac{8}{3\pi}\alpha_s(\sqrt{-t})}+o(\alpha_s^{1/2}(\sqrt{-t}))\,,$$
$$\alpha_{gg}(t)=1+\frac{12\,\ln 2}{\pi}\alpha_s(\sqrt{-t})+o(\alpha_s(\sqrt{-t}))\,,$$

where α_s is the QCD running coupling.

As well, the intercept of the leading Regge trajectory, pomeron, was calculated by Fadin and Lipatov and, also, Ciafaloni and Camici [4]:

$$\alpha_{gg}(0)=1+\frac{12\,\ln 2}{\pi}\alpha_s(\mu)\left(1-\frac{20}{\pi}\alpha_s(\mu)\right)+o(\alpha_s^2(\mu))\,.$$

In contrast to the asymptotic relations at high transfers, the last expression is explicitly not renorm-invariant, since it depends on an arbitrary renormalization scale μ. Certainly, though Regge trajectories as analytic (i.e. unique) continuations of resonance spectra should be renorm-invariant, they could be approximated by some renorm-noninvariant expressions (like in QED), but the proper choice of the renormalization scheme and scale should be physically motivated. Also, theoretical uncertainty determined by the neglected terms should be low enough for a possibility of practical use of the obtained renorm-noninvariant approximations under calculation of scattering amplitudes. At the current stage of the BFKL approach development, the theoretical uncertainty of the BFKL pomeron intercept value is rather high, and, thus, the BFKL method provides information only about asymptotic behavior of QCD Regge trajectories at ultra-high values of transferred momentum.

The second approach is the less-known Lovelace approach [5] which deals with the Bethe-Salpeter equation in some asymptotic regime, where we do not need reggeization of the partons composing the considered bound state. The main feature of this approach is exploitation of renorm-invariant kernels in the BS equation. As a result, we obtain renorm-invariant numbers for intercepts of Regge trajectories. In his paper [5] Lovelace considered asymptotically free ϕ_6^3-theory and found some infinite series of intercepts:

$$(\alpha_{\phi\phi}^{(k)}(0)+1)(\alpha_{\phi\phi}^{(k)}(0)+2)(\alpha_{\phi\phi}^{(k)}(0)+3)=\frac{16}{3(2k+1)}$$

with $\alpha_{\phi\phi}^{(0)}(0) \approx -0.06273$. In other two papers the analogous results for the gluon-gluon [6] and quark-antiquark [7] trajectories in QCD were obtained:

$$\alpha_{gg}^{(k)}(0)(\alpha_{gg}^{(k)}(0)+1)(\alpha_{gg}^{(k)}(0)+2) = \frac{24N_c}{(2k+1)(11N_c-2n_f)}\,,$$

$$\alpha_{\bar{q}q}^{(k)}(0) = \frac{9(N_c^2-1)}{(2k+1)N_c(11N_c-2n_f)} - 1$$

(if $N_c = 3$ and $n_f = 6$, then $\alpha_{gg}^{(0)}(0) \approx 0.7276$ and $\alpha_{\bar{q}q}^{(0)}(0) = 1/7$). Note, that in the Lovelace approach the intercepts of Regge trajectories do not depend on the coupling at all, what is a general consequence of renorm-invariance in massless field theories [8].

Unfortunately, due to technical difficulties, the leading gluon-gluon and quark-antiquark series (corresponding to the trajectories with the Kwiecinski and Kirschner-Lipatov asymptotical behavior) were not calculated, though in [6] there was proved the existence of some infinite series condensing to $\alpha_{gg}^{(\infty)}(0) = 1$ from above.

Thus, we have got information about quantitative behavior of the QCD leading Regge trajectories at very high momentum transfers only. This region of transferred momenta, however, gives a negligible contribution to diffractive cross-sections. In other words, at present moment, QCD does not provide any quantitative result which could be directly used under construction of phenomenological models of hadronic diffraction, though some models try to adopt qualitative QCD features.

Extraction of the pomeron intercept from the DIS data

Also, a question emerges, if there exists a possibility to extract some quantitative characteristics of QCD Regge trajectories in the scattering region directly from experimental data.

Such a possibility exists. It turns out that in Deep Inelastic Scattering there is a wide kinematical range where the $\gamma^* p$ total cross-sections can be well-described by a simple formula (below $W_0 \equiv 1$ GeV):

$$\sigma_{tot}^{\gamma^* p}(W^2, Q^2) \approx \beta(Q^2)\left(\frac{W}{W_0}\right)^{2\delta}\,,$$

where W is the invariant mass of the produced hadronic state and Q^2 is the incoming photon virtuality.

The value of δ does not depend on the photon virtuality. Such independence is principal, since only in this case δ can be associated with the intercept of the pomeron trajectory:

$$\alpha_{\mathrm{P}}(0) = 1 + \delta\,.$$

Extraction of δ from the DIS data [9] (in the kinematical range summarized in Tab. 1) yields $\frac{\chi^2_{min}}{N_{DoF}} \approx 0.977$ ($N_{DoF} = 574$) and $\delta = 0.303^{+0.057}_{-0.056}$ ($\Delta\left[\frac{\chi^2}{N_{DoF}}\right] = 1$).[1]

More accurate determination of the pomeron intercept could be a task for future lepton-hadron colliders.

Phenomenological models

The phenomenological models of the nucleon-nucleon elastic diffractive scattering, proposed before the TOTEM preliminary results had been published, could be divided into 2 groups:

[1] Three outlying points were excluded from the fitting procedure (1 point at $Q^2 = 35$ GeV^2, 1 point at $Q^2 = 350$ GeV^2, and 1 point at $Q^2 = 800$ GeV^2).

Set of data	$\beta_{min}(Q^2)$, mb	Number of points	χ^2_{min}
$Q^2 = 25$ GeV^2, 50 GeV $< W <$ 300 GeV	0.000254	24	26.1
$Q^2 = 35$ GeV^2, 50 GeV $< W <$ 300 GeV	0.000182	38	39.1
$Q^2 = 45$ GeV^2, 50 GeV $< W <$ 300 GeV	0.000139	25	23.0
$Q^2 = 60$ GeV^2, 50 GeV $< W <$ 300 GeV	0.000102	33	21.2
$Q^2 = 90$ GeV^2, 40 GeV $< W <$ 300 GeV	0.0000645	29	16.8
$Q^2 = 120$ GeV^2, 40 GeV $< W <$ 300 GeV	0.0000457	36	38.3
$Q^2 = 150$ GeV^2, 30 GeV $< W <$ 300 GeV	0.0000352	32	22.2
$Q^2 = 200$ GeV^2, 30 GeV $< W <$ 300 GeV	0.0000243	44	47.5
$Q^2 = 250$ GeV^2, 30 GeV $< W <$ 300 GeV	0.0000187	46	37.5
$Q^2 = 300$ GeV^2, 30 GeV $< W <$ 300 GeV	0.0000153	18	13.4
$Q^2 = 350$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.0000125	22	22.0
$Q^2 = 400$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.0000105	22	14.4
$Q^2 = 500$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.00000785	30	31.9
$Q^2 = 650$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.00000563	37	29.2
$Q^2 = 800$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.00000437	39	47.7
$Q^2 = 1000$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.00000324	20	21.3
$Q^2 = 1200$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.00000251	31	32.1
$Q^2 = 1500$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.00000189	24	22.2
$Q^2 = 2000$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.00000124	28	32.6
$Q^2 = 3000$ GeV^2, $\sqrt{3Q^2} < W <$ 300 GeV	0.00000076	17	22.1

Table 1: The quality of description of the DIS data at various Q^2 by the simple power formula.

the models exploiting the notion of reggeons [10, 11, 12, 13, 14, 17, 18, 19, 23, 24, 25, 26, 27] and the non-reggeon models [15, 16, 20, 21, 22].

In several reggeon models the eikonal representation of the scattering amplitude is used, where the eikonal is the sum of single-reggeon exchange terms with a supercritical pomeron (or pomerons) as a leading term [13, 14, 27]. Other reggeon models do not use the eikonal representation but introduce some complicated leading Regge singularities: the pomeron as double Regge pole in [10, 11, 12, 17, 23], and the so-called froissaron, the leading Regge cut, in [18, 19]. Also, these models take into account the contributions from secondary Regge poles.

There exists a separate subgroup of reggeon models [24, 25, 26] exploiting methods of the Reggeon Field Theory. The eikonal here is replaced by the so-called opacity which is the sum of not only single-reggeon-exchange terms but, also, multi-reggeon exchanges. Low-mass dissociation in the intermediate states is taken into account as well.

The non-reggeon phenomenological schemes could be divided into the models not appealing to QCD [15, 21] and the so-called QCD-inspired models [16, 20, 22]. Model [15] exploits general principles and the derivative dispersion relations as extra conditions. Model [21] is some variant of the quasi-potential approach. In [16] nucleon is considered to have an outer cloud of quark-antiquark pairs, an inner shell of baryonic charge, and a central quark bag containing valent quarks (small-angle scattering is due to overlapping of the outer clouds and the exchange by ω-meson). In the Dipole Cascade Model [20] nucleon is introduced as color dipole and interaction between hadrons at ultra high energies is presumed to be dominated by perturbative effects. Model [22] uses the eikonal composed of 3 terms which are called the contributions from the quark-quark, quark-gluon, and gluon-gluon interaction, though the corresponding expressions are not derived from QCD directly and contain numerous free parameters.

All the mentioned models give different predictions for the nucleon-nucleon total and dif-

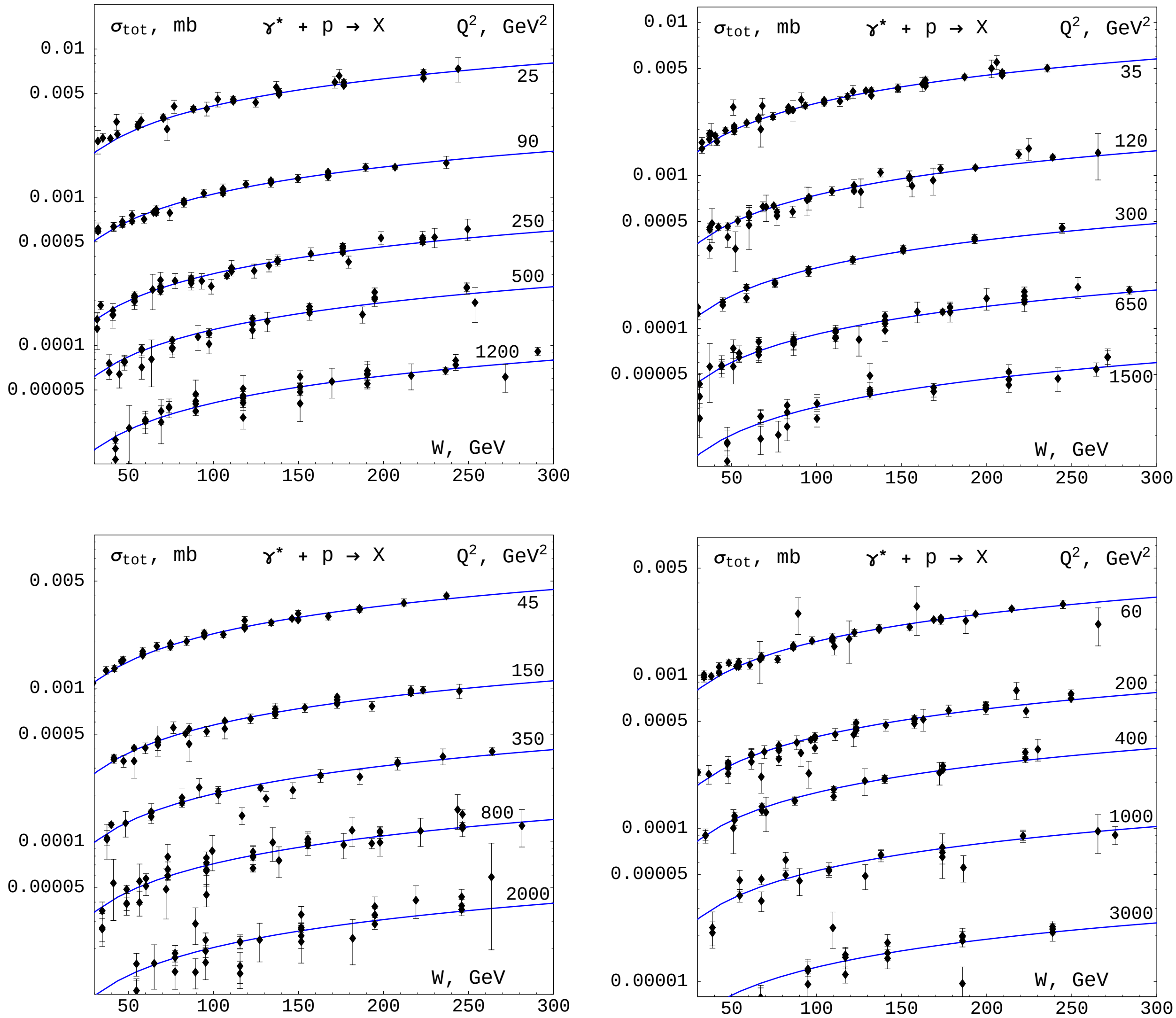

Figure 1: Description of the $\gamma^{*}p$ total cross-sections by the simple power formula with $\delta_{min} = 0.303$.

ferential cross-sections at the LHC.

Models vs. D0 and TOTEM

Comparison of the model predictions for the pp total cross-section with the value measured by the TOTEM Collaboration [28] reveals that some of the models may be judged as discriminated already at this stage (Tab. 2). Though others survive.

However, some theorists may consider deviations in several percents from experimental data to be not fatal for any model. In any case, measurement of total cross-section only is not enough for proper discrimination among the models.

Some models can be discriminated via comparison with the recently published D0 data on the $\bar{p}p$ differential cross-section [29] (Fig. 2).

In their turn, the TOTEM data on the pp differential cross-section [28, 30] reveal a very strong discriminative power (Fig. 3). Inversely, the predictive efficiency of all the considered

The Model	$\sigma_{tot}^{pp}(7\,TeV)$, mb
P. Desgrolard, M. Giffon, L.L. Jenkovszky, Z. Phys. C **55** (1992) 637	87 (*6 TeV*)
A. Donnachie, P.V. Landshoff, Phys. Lett. B **296** (1992) 227	91
P. Desgrolard, M. Giffon, E. Martynov, Eur. Phys. J. C **18** (2000) 359	95
V.A. Petrov, A.V. Prokudin, Eur. Phys. J. C **23** (2002) 135	97 ± 4
C. Bourrely, J. Soffer, T.T. Wu, Eur. Phys. J. C **28** (2003) 97	93
R.F. Avila, S.D. Campos, M.J. Menon, J. Montanha, Eur. Phys. J. C **47** (2006) 171	94
M.M. Islam, R.J. Luddy, A.V. Prokudin, Int. J. Mod. Phys. A **21** (2006) 1	97.5
E. Martynov, Phys. Rev. D **76** (2007) 074030	91
R.F. Avila, P. Gauron, B. Nicolescu, Eur. Phys. J. C **49** (2007) 581	108
E. Martynov, B. Nicolescu, Eur. Phys. J. C **56** (2008) 57	95
C. Flensburg, G. Gustafson, L. Lönnblad, Eur. Phys. J. C **60** (2009) 233	98 ± 9
P. Brogueira, J. Dias de Deus, J. Phys. G **37** (2010) 075006	110
M.M. Block, F. Halzen, Phys. Rev. D **83** (2011) 077901	95.5 ± 1
L.L. Jenkovszky, A.I. Lengyel, D.I. Lontkovskyi, Int. J. Mod. Phys. A **26** (2011) 4755	98 ± 1
E. Gotsman, E. Levin, U. Maor, Eur. Phys. J. C **71** (2011) 1553	91
M.G. Ryskin, A.D. Martin, V.A. Khoze, Eur. Phys. J. C **71** (2011) 1617	89
S. Ostapchenko, Phys. Rev. D **83** (2011) 014018	93
A. Godizov, Phys. Lett. B **703** (2011) 331	110
The TOTEM Collaboration, CERN-PH-EP-2012-239	98.58 ± 2.23

Table 2: Comparison of the model predictions for the pp total cross-section at $\sqrt{s} = 7$ TeV with the TOTEM result.

models turned out very weak. I mean the fact that many models give a nice description of differential cross-sections from the ISR to the Tevatron energies [31] (with the collision energy increase in several tens of times). But though the ratio of the LHC energy to the Tevatron energy is only about 4, we can observe a huge discrepancy between the model curves and the data (for some models — tens of percent, for others — several times).

Another question: what are the consequences of such a result for QCD? Is it falsified? Certainly, no. Only the phenomenological models are subjects of falsification but QCD is not. Such a situation takes place due to the fact that all the hadron diffraction models are not grounded on analytical derivations from QCD, though some of them use adopted QCD terminology.

In a year, since the TOTEM preliminary results had been published, we returned to the same stage as before the TOTEM measurements: there are numerous models [32] – [46] (very different by physical ground) which describe the TOTEM data more or less satisfactorily. But who can guarantee that such a simultaneous failure will not be reproduced after the forthcoming measurements at 14 TeV?

Conclusion

We need a deeper interrelation of phenomenological models with QCD. First of all, we need development of some non-perturbative techniques for calculation of Regge trajectories in the diffractive (large distance) domain of QCD.

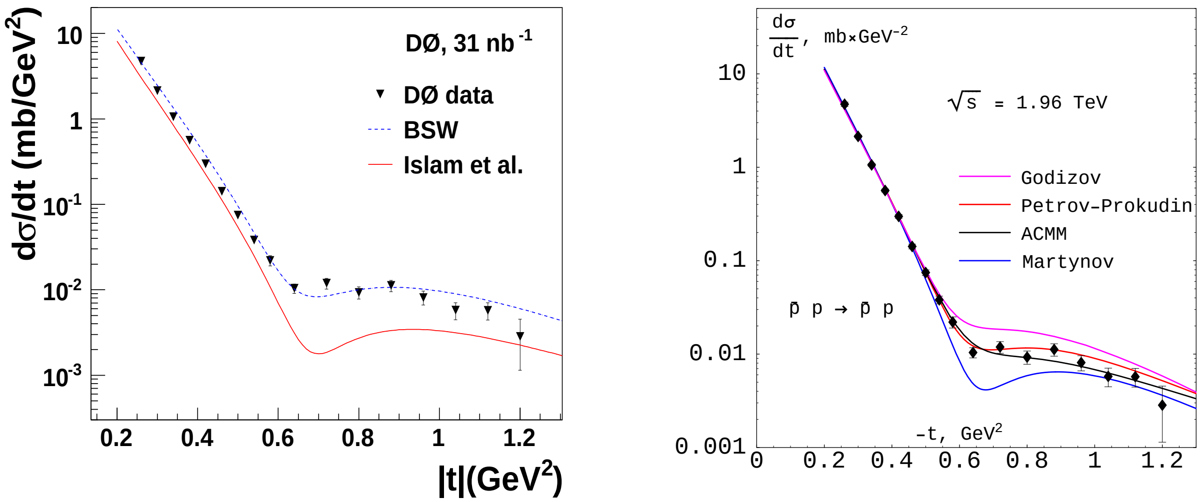

Figure 2: Comparison of the model predictions for the $\bar{p}p$ differential cross-section at $\sqrt{s} =$ 1.96 TeV with the D0 results (the left picture is taken from [29]).

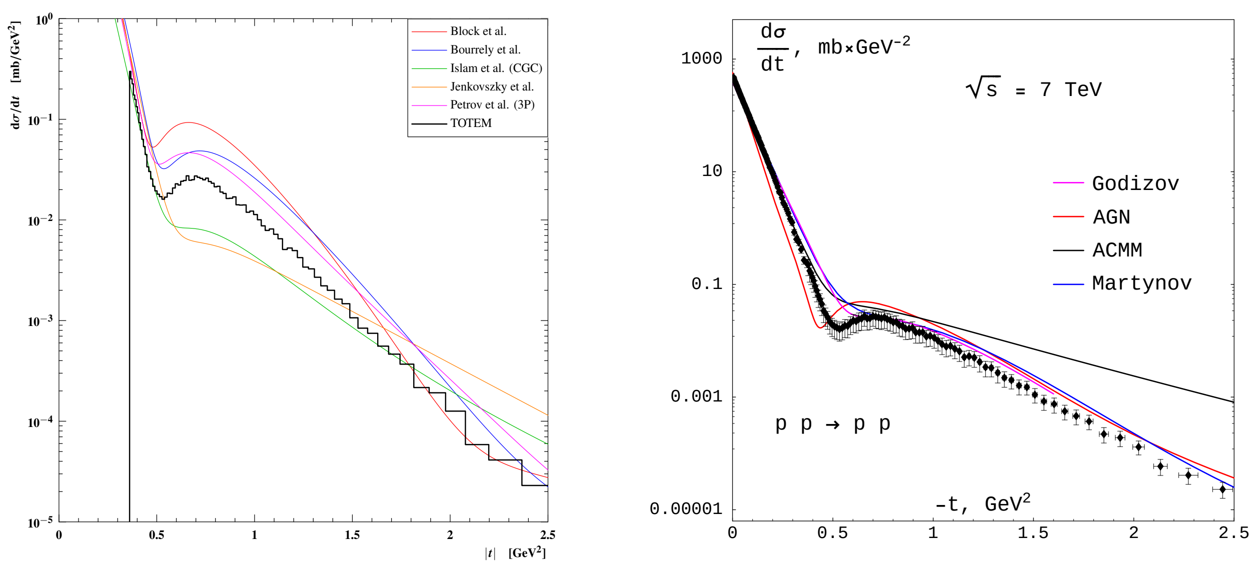

Figure 3: Comparison of the model predictions for the pp differential cross-section at $\sqrt{s} = 7$ TeV with the TOTEM results (the left picture is taken from [30]).

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H1 Collaboration (S. Aid *et al.*), Nucl.Phys. B **470** (1996) 3
ZEUS Collaboration (M. Derrick *et al.*), Z.Phys. C **72** (1996) 399
ZEUS Collaboration (J. Breitweg *et al.*), Eur.Phys.J. C **7** (1999) 609
H1 Collaboration (C. Adloff *et al.*), Eur.Phys.J. C **13** (2000) 609
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